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World Energy Projection System Plus: Industrial Module

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1. Introduction

Purpose of this Report

The World Industrial Model of the World Energy Projection System Plus (WEPS+) is an energy demand modeling system that projects industrial end-use sector energy consumption for 16 international regions. This report describes the version of the World Industrial Model (WIM) that was used to produce the industrial sector projections published in the *International Energy Outlook 2017* (*IEO2017*). WIM is one of 13 components of WEPS+ energy modeling system, but the industrial model can also be run as a separate, individual model. The WEPS+ is a modular system, consisting of separate energy models that communicate and work together through the overall system model. These models are each developed independently, but are designed with well-defined protocols for system communication and interactivity. The WEPS+ modeling system uses a shared database (the "restart" file) that allows all the models to communicate with each other when they are run in sequence over a number of iterations. The overall WEPS+ system uses an iterative solution technique that allows for simultaneous convergence of consumption and price to simulate market equilibrium.

This report documents the objectives, analytical approach and development of the WIM. The report catalogues and describes critical assumptions, computational methodology, parameter estimation techniques, and model source code. This document serves three purposes. First, it is a reference document providing a detailed description for model analysts, users, and the public. Second, it meets the legal requirement of the U.S. Energy Information Administration (EIA) to provide adequate documentation in support of its models (*Public Law 93-275, section 57.b.1*). Third, it facilitates continuity in model development by providing documentation from which energy analysts can undertake and analyze their own model enhancements, data updates, and parameter refinements for future projects.

Model Summary

The World Industrial Model of the World Energy Projection System Plus (WEPS+) projects energy use in the manufacturing, agricultural, construction and mining sectors for 18 energy sources in each of the 16 WEPS+ regions on an annual basis to 2050. The model primarily employs a stock-flow accounting framework that uses exogenously-specified output of specific industries as a proxy for productive capacity. In WIM, old capacity is assumed to be retired over time to make way for new capacity. This framework allows for the modeling of changes in energy efficiency over time. Exogenously-specified energy prices impact both the mechanisms of the stock-flow process and fuel switching projections over time.

Model Archival Citation

This documentation refers to the WEPS+ World Industrial Model, as archived for the *International Energy Outlook 2016* (*IEO2016*).

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Organization of This Report

Chapter 2 of this report discusses the purpose of WIM, the objectives and the analytical issues it addresses, the general types of activities and relationships it embodies, the primary input and output variables, and the relationship of the model to the other models in the WEPS+ system. Chapter 3 of the report describes the rationale behind the WIM, providing insights into further assumptions used in the model. Chapter 4 describes the model structure in more detail, including flowcharts, variables, and equations.

2. Model Purpose

Model Objectives

The primary objective of the WIM is to generate annual projections of industrial energy use for 2013 through 2050 for each of the 16 WEPS+ international regions. Projections are further broken out by fuel type, industry, and capacity vintage. As an integral component of the WEPS+ system, WIM provides consumption inputs to the various transformation and supply models of WEPS+ and contributes to the simulation of the overall energy supply and demand balance. The consumption inputs are also used by the Greenhouse Gases Model to calculate energy-related carbon dioxide emissions.

As part of the WEPS+ system, WIM operates at the level of the 16 WEPS+ world regions shown in Table 1. These regions consist of countries and country groupings within the broad divide of the Organization of Economic Cooperation and Development (OECD) membership. Multicountry regions (e.g. OECD Europe) are enumerated in Appendix A.

Table 1. Regional Coverage of the World Energy Projection System Plus Model

OECD Regions and index	Non-OECD Regions
1 United States	8 Russia
2 Canada	9 Non-OECD Europe and Eurasia
3 Mexico/Chile	10 China
4 OECD Europe	11 India
5 Japan	12 Other Non-OECD Asia
6 Australia/New Zealand	13 Middle East
7 South Korea	14 Africa
	15 Brazil
	16 Non-OECD Americas

Models Inputs and Outputs

Inputs

WIM uses macroeconomic and industrial price projections that are imported from the WEPS+ restart file. These inputs have been previously projected by the Macroeconomic Model and the transformation and supply models (Table 2). The macro index is not always the same as the industrial index in table 5. The following is the list of the industrial index first from 1 to 16 and the macro index second: 1 1; 2 2; 3 3; 4 4; 5 5; 6 6; 7 7; 8 8; 9 9; 10 10; 11 17; 12 18; 13 19; 14 20; 15; 21; and 16 12.

Table 2. WEPS+ Models that Provide Input to the World Industrial Model

World Industrial Model Input		Source
Gross domestic product		Macroeconomic Model
Outputs of specific industries		Macroeconomic Model
Industrial retail prices		
Energy Source Index number and name		
1	Motor gasoline	Refinery Model
2	Distillate	Refinery Model
3	Residual fuel	Refinery Model
4	Kerosene	Refinery Model
5	Liquid Petroleum Gas (LPG)	Refinery Model
8	Other petroleum	Refinery Model
9	Natural Gas	Natural Gas Model
10	Coal	Coal Model
11	Electricity	World Electricity Model
12	District heat	District Heat Model

The data series listed in Table 3 are also imported into the World Industrial Model from the WIMInput.xml file.

Table 3. Major Exogenous World Industrial Model Input Data Series

Source Input File	World Industrial Model Input
WIMInput.xml	Capacity retirement rates by industry and region
	Capacity retirement rate elasticities by industry and region
	Relative Energy Intensity (REI) coefficients by industry and region
	Technology Possibility Curve (TPC) growth rates for existing and new capacity by industry and region
	Technology Possibility Curve (TPC) price elasticities for existing and new capacity by industry and region

Outputs

WIM projects annual energy consumption by fuel and region. Upon completion of a model run, the projections are exported into the WEPS+ restart file for use by other models (Table 4).

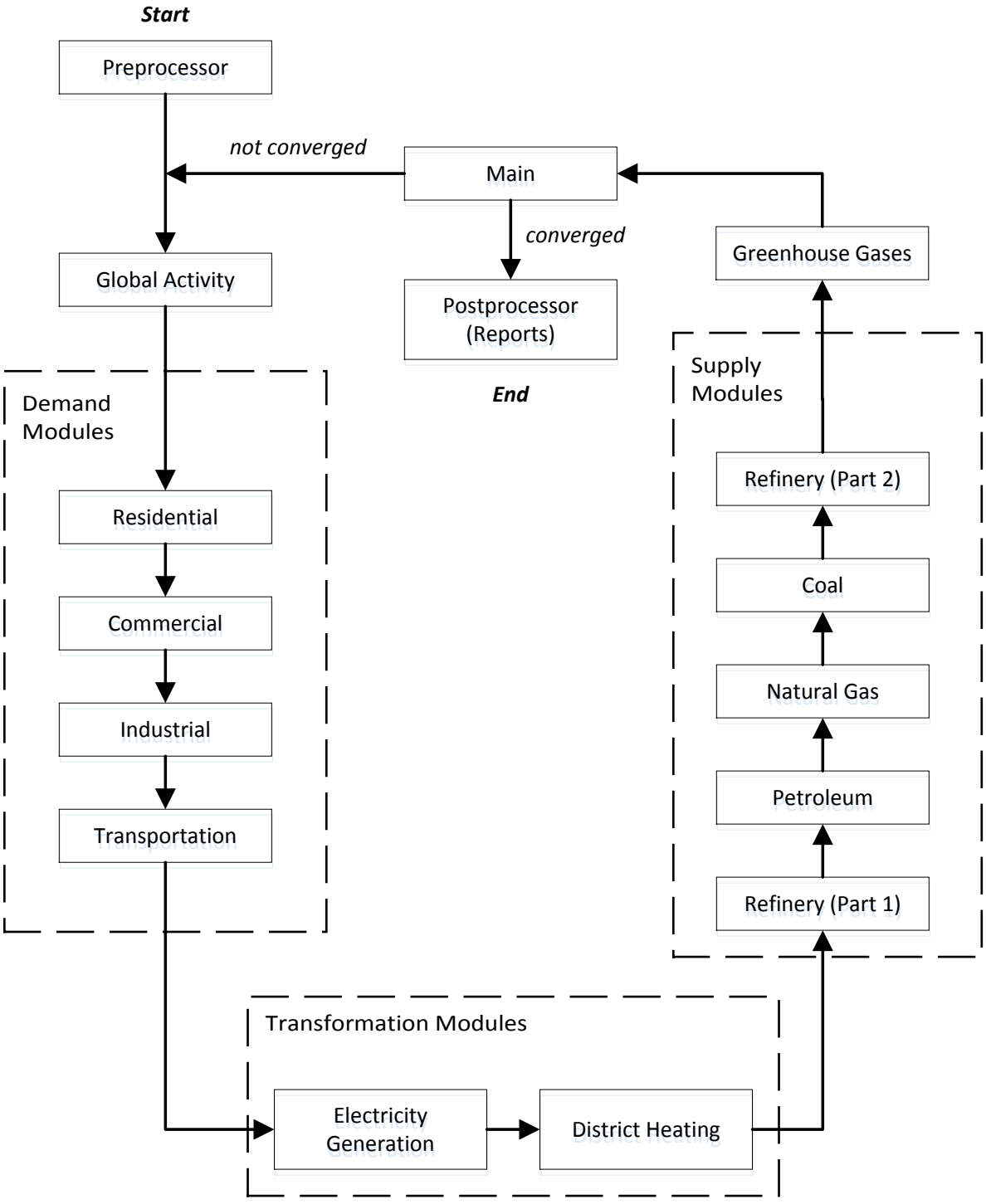
Table 4. WIM Model Output and the WEPS+ Models that Use Them

World Industrial Model Output	Destination
Petroleum consumption	Petroleum and Refinery Model
Motor gasoline	
Distillate	
Residual fuel	
Kerosene	
LPG	
Petroleum coke	
Sequestered petroleum	
Other petroleum	
Crude oil (direct use)	
Natural gas consumption	Natural Gas Model
Coal consumption	Coal Model
Electricity consumption	World Electricity Model
Heat consumption	District Heat Model
Waste consumption	Reporting
Biomass consumption	Reporting
Geothermal consumption	Reporting
Solar consumption	Reporting
Other renewable consumption	Reporting

Relationship to Other Models

WIM is an integral component of the WEPS+ system and depends upon other models in the system for some of its key inputs. In turn, the WIM provides projections of energy consumption that other models in the system depend upon for their key inputs (Figure 1). A summary description of the models, flows, and mechanics of the WEPS+ system used for the *IEO2016* report is available in a separate *Overview* documentation.

Figure 1. World Energy Projection System Plus (WEPS+) Model Sequence

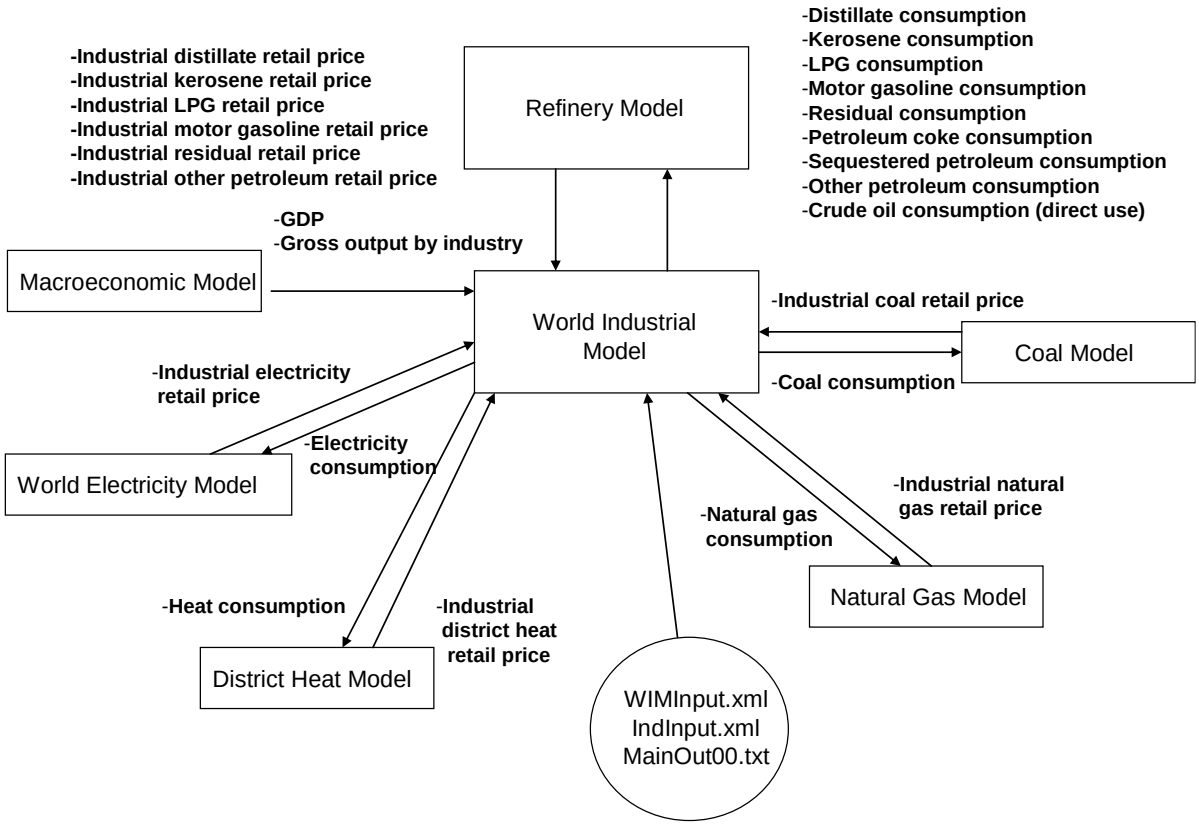


Through the system, WIM receives gross domestic product (GDP) projections and gross output projections for specific industries from the Macroeconomic Model (also called the Global Activity Model) and receives a variety of price projections from various supply and transformation models (Figure 2). In

turn, WIM provides consumption projections through the system back to the supply and transformation models.

Although WIM is an integral part of the WEPS+ system, it can also be easily run as a standalone model. In standalone mode, WIM inputs macroeconomic and price projections from the WEPS+ system "restart" file as created in a previous full-system run.

Figure 2. The World Industrial Model Relationship to Other WEPS+ Models



3. Model Rationale

Theoretical Approach

The World Industrial Model (WIM) projects energy use that is directly consumed as a fuel or as a feedstock by industrial processes and activities. WIM includes manufacturing industries along with non-manufacturing industries such as construction, agriculture, and extraction (mining). The model is used to project industrial consumption for 18 energy sources in each of the 16 WEPS regions to the year 2050. WIM is a structured, industry-level, stock/flow model that uses gross output from the Macroeconomic Model as its primary driver. The model also uses retail energy prices for 12 energy sources. These drivers are available to the World Industrial Model from the WEPS+ transformation and supply models through the shared restart file.

WIM projects energy consumption in eight separate industries that are considered energy intensive or distinctive with respect to energy consumption. Other Chemicals and Other Manufacturing are non-energy intensive. Table 5 shows the 12 industries.

Table 5. Industrial Sector Industries

WIM Industry Number	Industry	Representative Industries
1	Food	Food, beverage, and tobacco product manufacturing.
2	Paper	Paper manufacturing, printing and related support activities.
3	Basic Chemicals	Inorganic chemicals, organic chemicals (e.g. ethylene, propylene), resins, and agricultural chemicals. Includes chemical feedstock.
4	Refining	Petroleum refineries and coal products manufacturing to include coal and natural gas used as feedstock.
5	Iron and Steel	Iron and steel manufacturing, including coke ovens.
6	Non-Ferrous Metals	Primarily aluminum and other non-ferrous metals such as copper, zinc, and tin.
7	Non-Metallic Minerals	Primarily cement and other nonmetallic minerals such as glass, lime, gypsum, and clay products.
8	Other	All other manufacturing.
9	Other Chemicals	Pharmaceuticals, medicinal and botanical, paint and coatings, adhesives, detergents, and other miscellaneous chemical products. Includes chemical feedstock.
10	Agriculture	Agriculture, forestry, and fishing.
11	Oil Extraction	Oil and gas extraction
13	Other Extraction	Metallic and non-metallic mining such as gold and lithium.
14	Motor Vehicles	Transportation equipment - motor vehicles and aircraft - manufacturing.
15	OMBD	Other metal based durables manufacturing, computer and electronic products, machinery, and electrical equipment appliances and components.
16	Construction	Buildings (residential and commercial) construction,, heavy and civil engineering construction, industrial construction, and specialty trade contractors.

WIM uses a stock/flow approach to estimate productive capacity needed to satisfy demand. It uses an existing stock of productive capacity for each industry in the base year. In subsequent projection years, the model estimates additional productive capacity needed to satisfy demand. The model calculates the change in productive capacity for three vintage categories:

Remaining Original Capacity (Existing Vintage). The model uses a retirement rate to determine how much of the original existing capacity remains (was not retired). The "original existing capacity" is simply the historical base year industrial capacity (the base year was 2012 for the *IEO2016*), which is based on

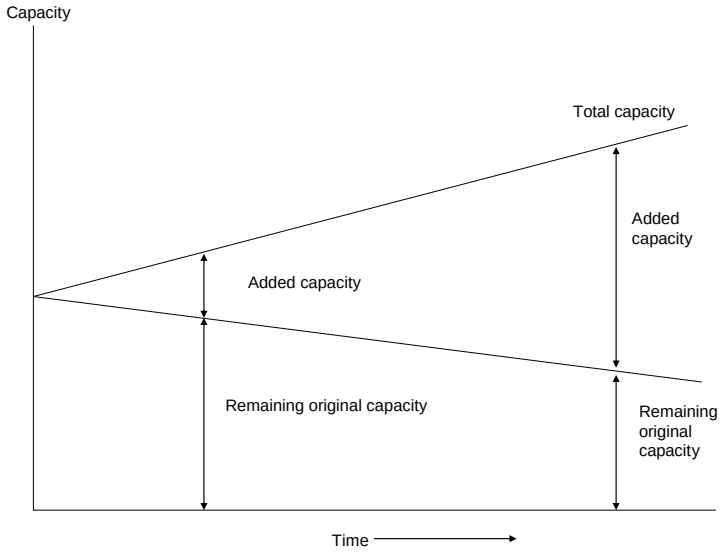
industrial output. The base year industrial energy intensity is computed from the original existing capacity.

Added Capacity (New Vintage) includes two types:

- Previously Added Capacity. The model projects changes in energy intensity (largely due to "housekeeping") and changes in the fuel mix for the cumulative amount of previously added capacity. "Previously added capacity" is the amount of capacity added between the base year up to the current model year.
- New Capacity. For the current model year, the model determines the amount of new capacity added by subtracting the previously available capacity (original plus previously added) from the total required capacity, given the projected growth in gross output for the industry. The model projects the energy intensity for this new capacity, which is generally much better than that for the overall stock. A new fuel mix for the new capacity is also determined.

Total energy consumption in each vintage category is computed as gross output multiplied by energy intensity. Consumption of each fuel in each vintage category is computed as total energy consumption multiplied by the vintage-category fuel share. As discussed below, these fuel shares can be fixed or allowed to vary depending on consumption trends and price changes. The overall total industry consumption by fuel is calculated as the sum of consumption over the vintage categories. This approach takes into account the growth in output by industry and the rate at which new, typically more efficient, capacity is added. Over time, in rapidly growing industries, the new added capacity becomes a much more important determinant of energy consumption than in slowly growing industries. A simplified representation of the stock/flow approach is shown in Figure 3.

Figure 3. Illustration of Stock/Flow Industrial Capacity Changes Over Time



WIM requires a significant amount of initial data along with a number of projection coefficients. The data used to project industrial sector consumption is drawn from the International Energy Agency's *Energy Statistics and Balances of OECD and non-OECD Countries* and calibrated for consistency with historical international energy data available from the Energy Information Administration. The coefficients for the *IEO2016* were based on comparisons of existing industrial energy intensities among world regions, taking into account the efficiency discrepancies between the more industrialized regions and the developing regions. Analysts used these comparisons, along with expert judgment, to estimate parameters such as regional energy intensities, capacity retirement rates, rates of change in intensity for remaining and added capacity, and fuel price elasticities.

Model Assumptions

WIM projects industrial energy consumption based upon numerous assumptions. The assumptions most critical to the projections include the following:

- Economic output is a suitable proxy for productive capacity, i.e., utilized capacity.[1]
- Energy intensity of economic output is assumed to be inversely related to energy efficiency.
- Energy efficiency of new productive capacity will be better than energy efficiency of old productive capacity.
- Energy efficiency of economic output improves over time.
- Energy prices impact the retirement rate of capital, the rate of efficiency change, and the choice of fuels. Fuel switching is considered by region with variations by region.

[1] The model does not explicitly consider capacity utilization. However, if there is a decline in industrial output, followed by a subsequent increase, then the model will logically assume that the capacity "lost" during the downturn was just idled, and that this idled capacity is to brought back "on line" first before adding new (more energy efficient) capacity when industrial output starts to ramp up again.

4. Model Structure

Structural Overview

The main purpose of the World Industrial Model is to project annual industrial sector energy consumption, by region and fuel type, for 2015 through 2050. The industrial energy consumption calculations incorporate inputs that reflect analysts' expectations of the state of technology in each region. Consumption is estimated for each of the 16 WEPS+ regions for 18 energy sources (motor gasoline, distillate fuel, residual fuel, kerosene, liquefied petroleum gas including feedstocks, petroleum coke, sequestered petroleum, other petroleum use including feedstocks, direct use crude oil, natural gas including feedstocks, coal including coking coal, electricity, heat, waste, biomass, geothermal, solar, and other renewables).

The basic structure of the World Industrial Model is illustrated in Figure 4. A call from the WEPS+ model interface to the WIM begins the import of information from the restart file and the other WEPS+ models needed to complete the projection calculations.

Next, a call from the main Industrial Model initiates the Ind subroutine (Figure 6), which uses data from the WIMInput.xml file to complete its calculations. To read in this information, the Ind subroutine calls the ReadXML subroutine (Figure 7), which assigns values from the WIMInput.xml file to local variables in the model. This file contains analyst-developed inputs for fuel switching fractions, price elasticities, technology possibility curve growth rates, and relative energy intensity coefficients.

After the ReadXML subroutine executes, the Ind subroutine begins to compute industrial capacity and consumption projections by industry, fuel, and region. First, GDP and industrial price indices are computed across the projection period, by region and fuel, and adjusted for a high world oil price case scenario. Next, an overall industrial quantity index is calculated as the product of the historical consumption of fuels and the GDP and industrial price indices using benchmark factors for each region and industry.

The main Industrial Model then calls the GetGO subroutine (Figure 8) to read in macroeconomic data from an input file and compute the economic growth rate. After calculating the projected growth over the projection period, this subroutine relates overall economic growth of consumption to growth of GDP. It uses a dollar-to-physical-unit growth rate to convert dollars of output to industrial gross output, accounting for inflation and technological advancements over time. This information is then written into an output file. Next, the InitCalc subroutine is called to set up the initial gross outputs, consumption vintages, and intensity vintages. Finally, a weighted average price for each industry is calculated, which is related to historical price indices.

The bulk of the calculations in the Ind subroutine are made in the CalcMan subroutine (Figure 9). CalcMan uses the data in the WIMInput.xml subroutine to estimate the amount of existing, added, new, and total capacities for each fuel, industry, and region. First, consumption growth for each fuel is calculated by region and industry using a formula that assumes a continuous growth rate. Then, a switching fraction and price elasticity determines how much fuel is switched because of price changes throughout the projection period. The totals for each fuel are then calibrated to agree with the

projected total fuel usage in each region and industry through 2050. Then, the AddUp subroutine is called to aggregate the quantities calculated in the previous subroutines and export reports. Several output files are generated and returned to the main World Industrial Model routine. After the Ind subroutine has executed, the WriteRestart subroutine is called to send the output from the WIM to the restart file that will be used in future iterations of WEPS+.

Flow Diagrams

Figure 4 is the flow chart for subroutines, input files, and output files called by the industrial model. Figure 5 through Figure 9 are flow charts that provide more specific details. Figure 4 three subroutines noted with (not IND) are called but are high level subroutines not in the industrial model. The code names for the output and input file write and read statements are listed below:

- IN-Debug.txt (10)
- AllOut.txt (8)
- IN-QtyPrc.txt (31)
- IN-GOOutA.txt (30)
- IN-GOOutB.txt (31)
- IN-RptInit.txt (30)
- IN-MoreDetail.txt (60)
- IN-Detail.txt (25)
- IN-Detail.csv (26)
- ChinaSngIeo2015.xml (IndSNG)
- IndInput-2010_07_25_JH.xml (indXML)
- WIMInput.xml (WIMXML)
- Ind_subsector_con.txt (IndHist)

Figure 4. Flow Chart for the Industrial Model Subroutines

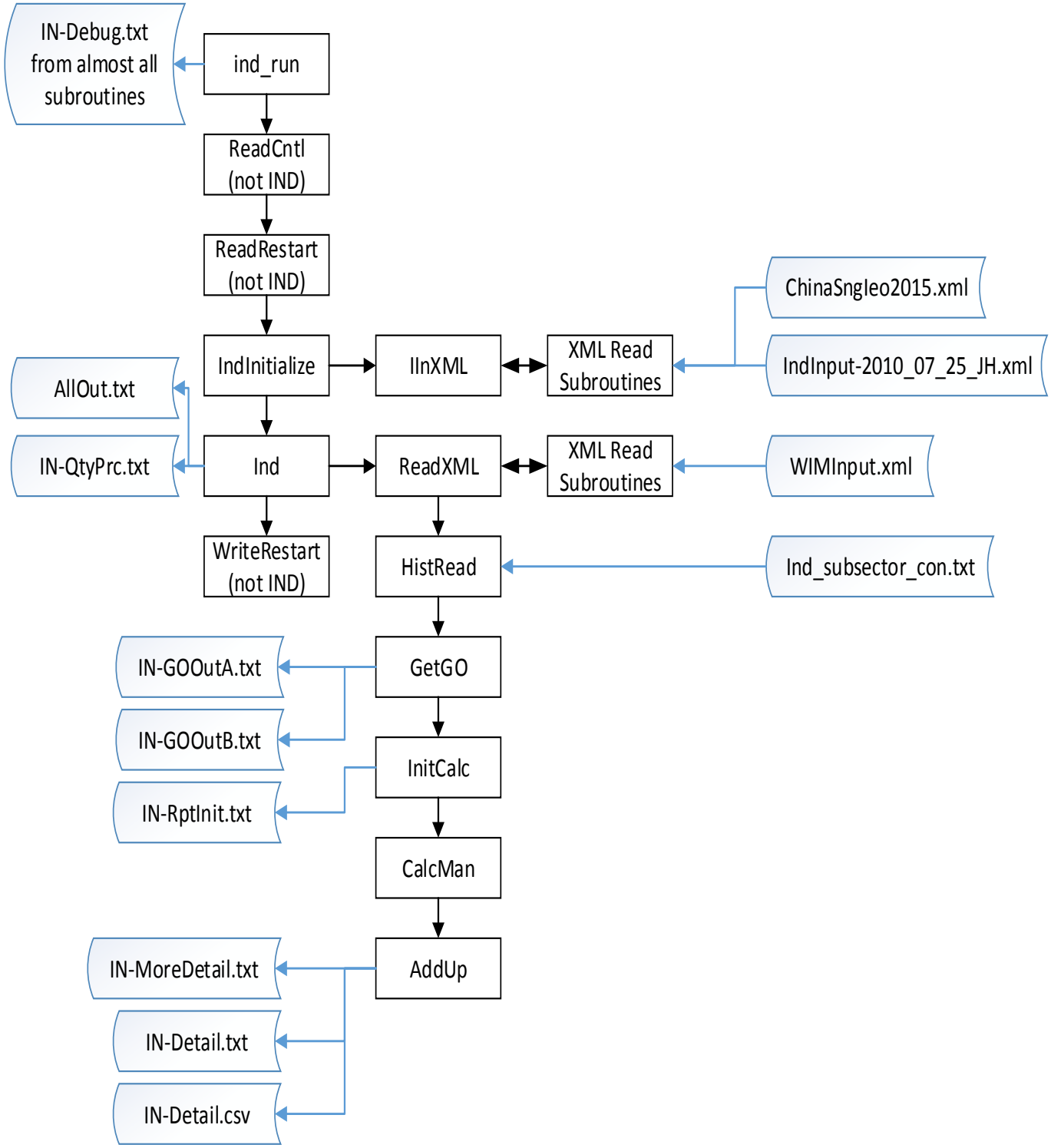


Figure 5. Flowchart for the Industrial Model

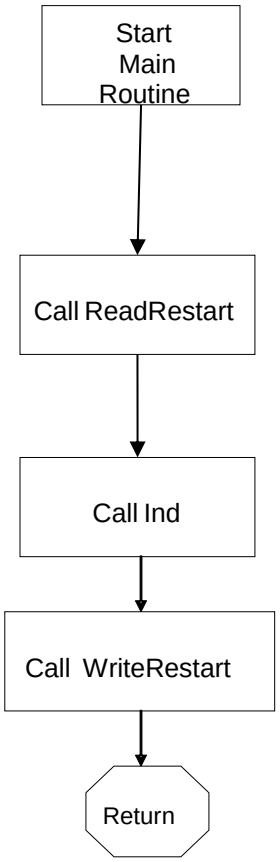


Figure 6. Flowchart for the Ind Subroutine

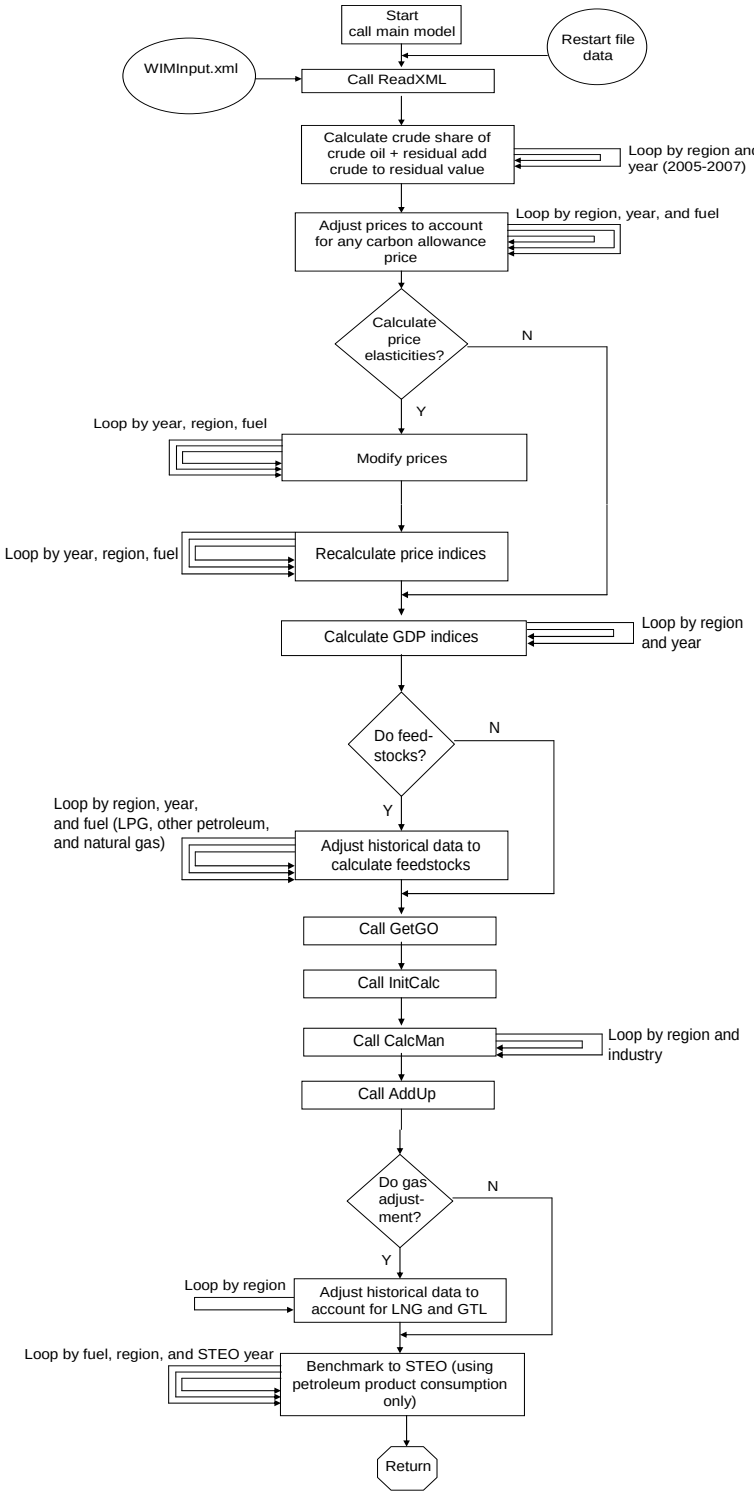


Figure 7. Flowchart for the ReadXML Subroutine

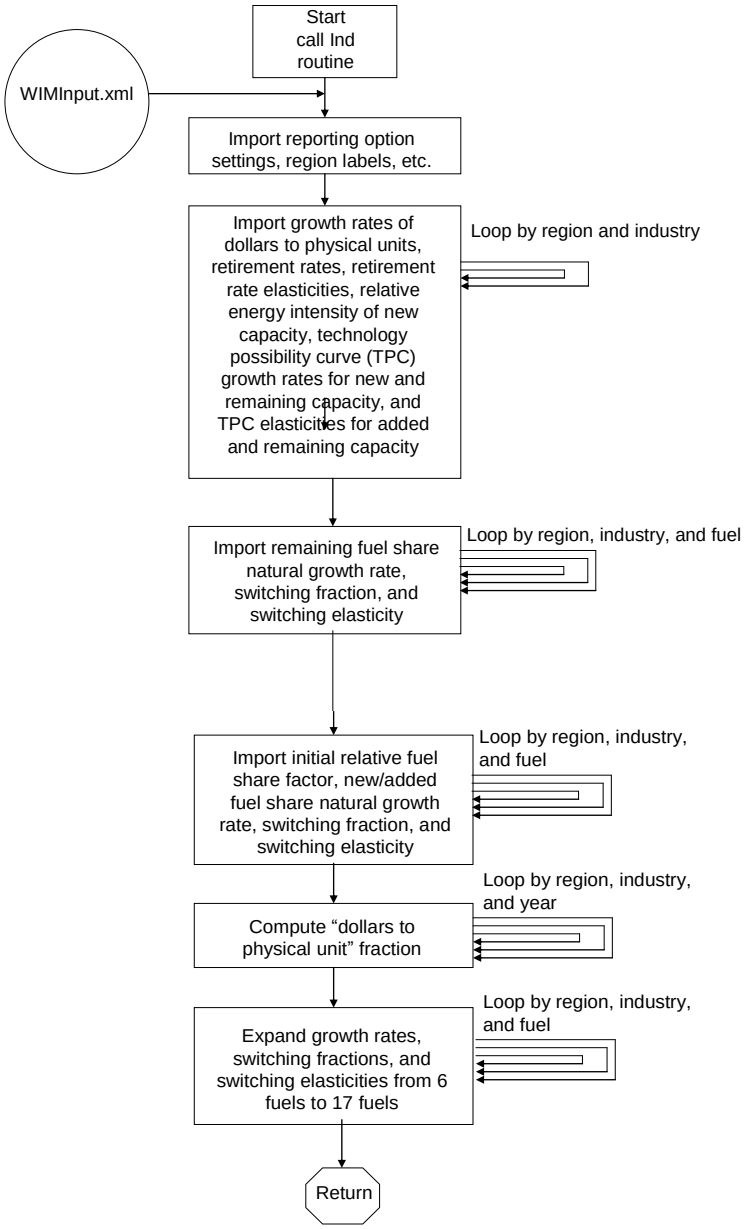


Figure 8. Flowchart for the GetGO Subroutine

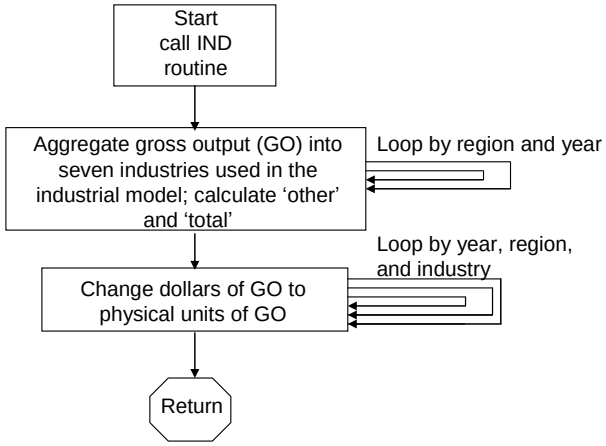
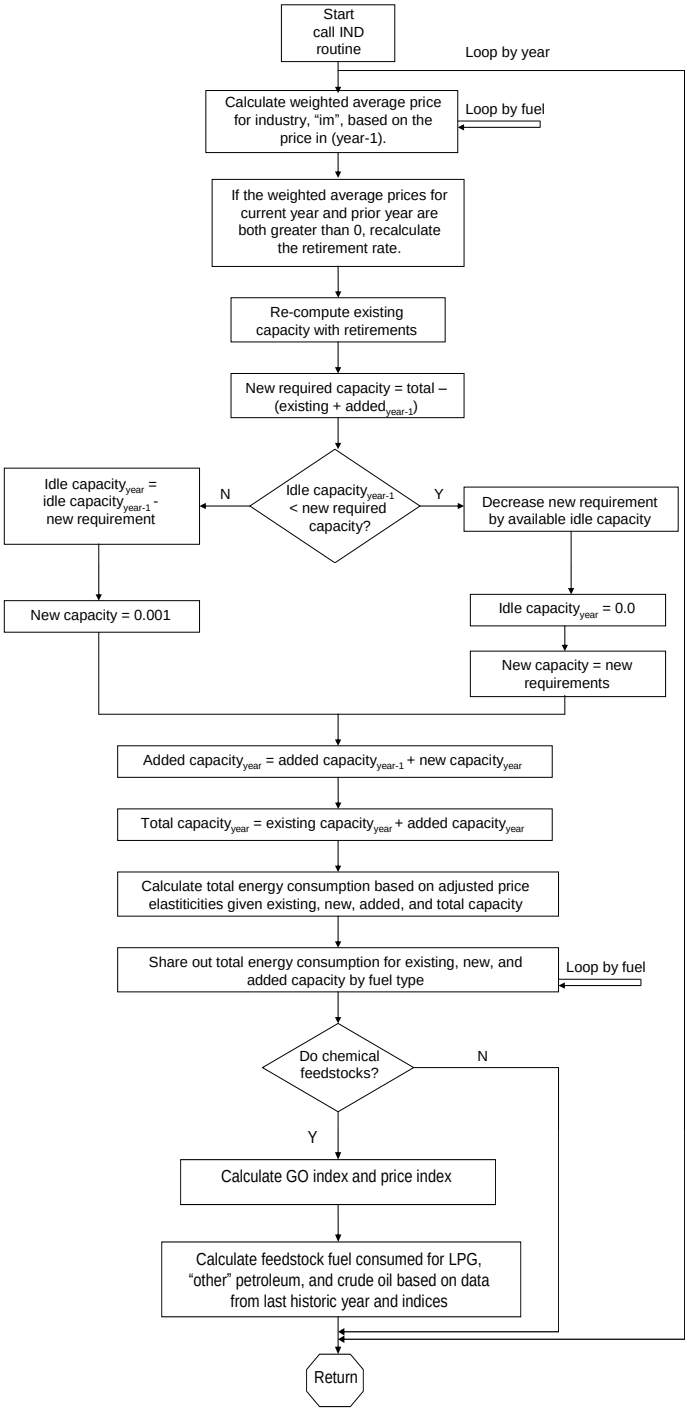


Figure 9. Flowchart for the CalcMan Subroutine



Key Computations

Gross Output

The baseline for the World Industrial Model projections consists of set of gross output projections for specific industries through 2050 for each of the 16 regions from the restart file. These values are generated by the macroeconomic model for specific industries, some of which are aggregates of others. For the WIM there are aggregates to the twelve industry categories that are used in the model:

1. Food
2. Paper
3. Basic Chemicals
4. Refining
5. Iron and Steel
6. Non-Ferrous Metals (primarily aluminum)
7. Non-Metallic Minerals (primarily cement)
8. Other Manufacturing
9. Other Chemicals
10. Agriculture
11. Oil Extraction
12. Coal Extraction
13. Oth Extraction (other mining)
14. Motor Vehicles (transportation equipment motor vehicles and aircraft)
15. OMBD (Other Metal Based Durables)
16. Construction

Vintaging of Gross Output as a Proxy for Capacity

WIM is a stock/flow model that keeps track of simple vintaging of the capacity for each industry and in each region as illustrated in Figure 3. Since there is no direct measure of capacity or a measure of physical output, the model uses the dollar value of gross output as a proxy for capacity. In WIM, gross output is exogenously specified.

To estimate change in output across the projection period the model adds output from new capacity, and subtracts output from existing capacity that is being retired. The "added gross output" is then computed as the sum of gross output from newly installed capacity and the amount of gross output from capacity added in the previous projection year. Finally, the existing/remaining gross output is added to the "added gross output" to calculate the projected "total gross output." The WIM does not have an exogenously supplied "new gross output" projection series. As a result, it is necessary to indirectly account for the new output for each projection year.

First, the model imports the estimated amount of total gross output, $TGrOut(x,y,r)$, for each industry x, forecast year y, and region r. Total gross output is the sum of existing gross output for the current year and added gross output for the current year. Existing gross output for this year is equal to last year's existing gross output adjusted by a retirement rate:

$$TGrOut(x,r,y) = EGrOut(x,r,y) + AGrOut(x,r,y) \quad (1)$$

$$EGrOut(x,r,y) = EGrOut(x,r,y-1) * \big(1.0 - RetRate(x,r,y)\big), \quad (2)$$

where

$TGrOut(x,r,y)$ = total gross output for industry x, region r, and year y

$EGrOut(x,r,y)$ = existing gross output for industry x, region r, and year y

$AGrOut(x,r,y)$ = added gross output for industry x, region r, and year y

$RetRate(x,r,y)$ = capacity retirement rate for industry x, region r, and year y

The model then calculates the new gross output added in year y by subtracting the total gross output for year $y-1$ from the total gross output for year y.

$$NGrOut(x,y,r) = TGrOut(x,r,y) - TGrOut(x,r,y-1) \quad (3)$$

$$= TGrOut(x,r,y) - [EGrOut(x,r,y-1) + AGrOut(x,r,y-1)], \quad (4)$$

where

$NGrOut(x,r,y)$ = new gross output for industry x, region r, and year y

Finally, the added gross output for the current projection year is calculated as the sum of the added gross output from the previous year plus the new gross output:

$$AGrOut(x,r,y) = AGrOut(x,r,y-1) + NGrOut(x,r,y) \quad (5)$$

Retirement Rate Adjustment

The retirement rate is adjusted over time based on the change in the weighted average energy price $WaPrc(x,r,y)$ and the exogenous retirement price elasticity for that industry and region. The relationship between the change in the retirement rate and prices is given by

$$\ln\left(\frac{RetRate(x,r,y)}{RetRate(x,r,y-1)}\right) = RetElas(x,r) * \ln\left(\frac{WaPrc(x,r,y)}{WaPrc(x,r,y-1)}\right) \quad (6)$$

where

$RetRate(x,r,y)$ = retirement rate for industry x, region r, and year y

$RetElas(x,r)$ = exogenous retirement elasticity with respect to changes in price for industry x, region r, and year y, which is read in from the WIMInput.xml file

$WaPrc(x,r,y)$ = weighted average prices, weighted by fuel, for industry x, region r, and year y

Energy Intensity and Consumption by Vintage

The industrial model projects the energy intensity (of consumption) for each stock/flow vintage of gross output for each industry and each region. Intensity is defined as the total amount of energy consumption for a specific industry in a specific region divided by the corresponding gross output. The intensity in each vintage category is the total consumption for that category divided by its gross output. The model estimates intensity in order to estimate consumption – it does not use consumption estimates to calculate intensity.

Initial Energy Intensity for the Existing Vintage

The model begins with the total consumption and the initial total gross output for each industry and region in the most recent historical year for which data are available. It uses the historical consumption and output data to calculate the energy intensity in the last historical year or initial year ($yi = 2012$) for each industry in each region. This is the starting point for the energy intensity that changes over the projection period.

$$EFmIntF(x,r,yi) = \frac{EFmQty(x,r,yi)}{EGrOut(x,r,yi)} \quad (7)$$

where

$EFmIntF(x,r,yi)$ = energy intensity for industry x, region r, and initial year yi

$EFmQtyF(x,r,yi)$ = quantity of energy consumed for industry x, region r, and initial year yi

$EGrOut(x,r,yi)$ = gross output of industry x, region r, and initial year yi

Energy Intensity by Vintage

The energy intensities for the existing and new vintage change over time for reasons such as technology upgrades, process changes, fuel substitution. For each industry and region, this process is represented by a coefficient that provides a growth rate trend factor, which is exogenous.

Existing Vintage

The energy intensities for the existing vintage changes over time for reasons such as technology upgrades, process changes, fuel substitution. For each industry and region, this process is represented by a coefficient that provides a growth rate trend factor, which is exogenous. The change in energy intensity for each year is given by

$$EFmIntF_1(x,r,y) = EFmIntF_1(x,r,y-1) * (1 + TPCRemGR(x,r)) \quad (8)$$

where:

$EFmIntF_1(x,r,y)$ = unadjusted measure of intensity by industry x,region r, and year y for the existing vintage

$TPCRemGR(x,r)$ = Technology Possibility Curve (TPC) growth rate for all fuels by industry x and region r for the existing vintage

Changing energy prices may also affect energy intensity. WIM models this process using a simple price elasticity:

$$PERFac(x,r,y) = \exp\left[TPCRemElas(x,r) * \ln\left(\frac{WaPrc(x,r,y)}{WaPrc(x,r,y-1)}\right)\right] \quad (9)$$

where

$PERFac(x,r,y)$ = TPC adjustment factor with respect to price for industry x,region r, and year y for the existing vintage

$TPCRemElas(x,r)$ = elasticity of the TPC with respect to price, for industry x and region r, which is exogenous and read in from the WIMINput file

$WaPrc(x,r,y)$ = weighted average fuel price, for industry x, region r, and year y, summed across all fuels

Multiplying the TPC adjustment factor by unadjusted energy intensity yields adjusted energy intensity for the existing vintage, and energy intensity is multiplied by existing gross output to yield energy consumption for the existing vintage:

$$EFmIntF(x,r,y) = PERFac(x,r,y) * EFmIntF_1(f,x,r,y) \quad (10)$$

$$EFmQtyF(x,r,y) = EGrOut(x,r,y)^* EFmIntF(x,r,y). \quad (11)$$

where

$EFmIntF(x,r,y)$ = adjusted energy intensity for industry x,region r, and year y for the existing vintage

$EFmQtyF(x,r,y)$ = total energy consumption for industry x,region r, and year y for the existing vintage

New Vintage

Over the course of the projection period, the model calculates the intensity for new capacity by multiplying the previous measure of intensity by a Relative Energy Intensity (REI) coefficient. The REI coefficient, $REINew(x,r)$, is exogenously specified for each industry x and region r. Energy Intensity for the initial year first year yi is as follows:

$$NFmIntF(x,r,yi) = EFmIntF(x,r,yi)^* REINew(x,r) \quad (12)$$

where

$NFmIntF(x,r,yi)$ = energy intensity for industry x in year yi and region r for the new vintage

$EFmIntF(x,r,yi)$ == energy intensity for industry x, region r, and initial year yi

$REINew(x,r)$ = energy Intensity for industry x and region r of new vintage equipment relative to existing vintage, which is read in from WIMInput.xml

The energy intensities for the new capacity change over time for reasons such as technology upgrades, process changes, and fuel substitution. For each industry and region, this process is represented by a coefficient that provides a growth rate trend factor, which is exogenous. Energy intensity for years after the initial year yi are as follows:

$$NFmIntF(x,r,y) = NFmIntF(x,r,y-1) * (1 + TPCNewGR(x,r)) \quad (13)$$

$$NFmQtyF(x,r,y) = NGrOut(x,r,y) * NFmIntF(x,r,y) \quad (14)$$

where

$NFmIntF(x,r,y)$ = energy intensity for industry x in year $y > yi$ and region r for the new vintage

$TPCNewGR(x,r)$ = TPC growth rate for all fuels by industry x and region r for the new vintage

$NFmQtyF(x,r,y)$ = total energy consumption industry x,region r, and year $y > yi$ for the new vintage;

$NGrOut(x,r,y)$ is as defined on page 24.

Energy consumption for the new vintage is the energy consumption of total added capacity, which is the sum of energy consumption of previously added capacity and new capacity. In subsequent years, energy consumption for added capacity is adjusted by a TPC adjustment parameter similar to $PERFac(x,r,y)$:

$$AFmQtyF_1(x,r,y) = AFmQtyF(x,r,y-1) + NFmQtyF(x,r,y) \quad (15)$$

$$AFmQty(x,r,y) = PEAFac(x,r,y) * AFmQtyF_1(x,r,y) \quad (16)$$

where

$AFmQtyF(x,r,y)$ = adjusted total energy consumption for industry x,region r, and year y for the new vintage

$AFmQtyF_1(x,r,y)$ = unadjusted total energy consumption industry x, region r, and year y for the new vintage

$NFmQtyF(x,r,y)$ = energy consumption of new capacity for industry x,region r, and year y

$PEAFac(x,r,y)$ = $\exp\left[TPCAddElas(x,r) * \ln\left(\frac{WaPrc(x,r,y)}{WaPrc(x,r,y-1)}\right)\right]$
TPC adjustment parameter with respect to price for industry x,region

r, and year y for the new vintage, analogous to $PERFac(x,r,y)$, which is defined on page 26.

Total energy Consumption and Intensity

The intensity and consumption forecast for the overall total capacity are determined through summing total energy use for added and existing capacity and dividing by total gross output:

$$TFmQtyF(x,y,r) = EFmQtyF(x,y,r) + AFmQtyF(x,y,r) \quad (17)$$

$$TFmIntF(x,r,y) = \frac{TFmQty(x,r,y)}{TGrOut(x,r,y)} \quad (18)$$

where

$TFmQtyF(x,r,y)$ = total energy consumption for industry x,region r, and year y

$TFmIntF(x,r,y)$ = energy intensity for industry x,region r, and year y for the new vintage

$TGrOut(x,r,y)$ = total gross output for industry x, region r, and year y

Consumption by Fuel

The WIM first computes total energy consumption by industry and region for each vintage category, as described above. Then it allocates the total consumption projections to the fuels used in the industrial sector. Although there are 17 fuel types represented in the WIM, the allocation algorithm is applied using six broad fuel categories. This is done to facilitate share specification. The share coefficients for percent fuel for these six fuel categories are later used for all of the subcategories. The six fuel categories are as follows:

- Petroleum (the sum of eight petroleum products)
- Natural Gas
- Coal
- Electricity
- District Heating
- Renewable (the sum of five renewable categories)

The ability to switch fuels is considered to be limited within these 6 groups. The switching on one petroleum fuel for another (say diesel for gasoline) would in most cases not make sense given that a) their prices are positively correlated, therefore savings possibilities are limited; and b) any savings does not justify the investment cost of switching, for example, from a diesel generator to one that runs on gasoline. The fuel sharing estimation methodology is also applied at the vintage category level, because, while the existing/remaining fuel shares are fairly fixed, the added fuel shares can differ as new capacity is installed.

Fuel Shares and Consumption by Fuel for the Existing Vintage

The existing vintage fuel shares ($EFShr$) are first set to their values in the initial projection year and then updated for subsequent years, based on relative energy consumption changes in each industry. Shares are calculated for each fuel:

$$EFShr(f,x,r,y) = \frac{EFmQty(f,x,r,y-1)}{EFmQtyF(x,r,y-1)} \quad (19)$$

$$EFmQty_1(f,x,r,y) = EFShr(f,x,r,y) * EFmQtyF(x,r,y) \quad (20)$$

where

$EFShr(f,x,r,y)$ = fuel share for fuel f industry x,region r, and year y for the existing vintage

$EFmQty(f,x,r,y)$ = adjusted quantity consumed of fuel f for industry x, region r, and year y for the existing vintage

$EFmQtyF(x,r,y)$ = total energy consumption for industry x,region r, and year y for the existing vintage

$EFmQty_1(f,x,r,y)$ = unadjusted quantity consumed of fuel f for industry x, region r, and year y for the existing vintage

As each individual share is updated, the sum of the shares may deviate from 1, making it necessary to renormalize the shares. The changes in shares may differ from the changes in actual consumption volume. For example, even though a fuel has a positive growth rate, its normalized share might decrease because other fuels have higher growth rates. Also, fuels with larger initial shares are rising from a larger base value, so minor changes in their shares may represent large consumption changes.

Fuel shares are also affected by fuel substitution in response to changes in relative prices. This is represented through a fuel switching algorithm that uses industrial sector fuel substitution elasticities. First, the relative price changes from the previous model year to the current model year are computed for all fuels $f \in F = \{c\ (coal),\ g\ (natural\ gas), o\ (oil), e\ (electricity)\}$. (All liquid fuels are grouped into the "oil" category). The elasticity $\eta_{f,l}$ represents the price elasticity of fuel f with respect to the price of fuel $l \in F$. Elasticities for each region are obtained or estimated directly from the 2009 World Bank report on interfuel substitution. The equation below computes the relative changes in the fuel mix. The fuel levels are then normalized so that the pre-switching and post-switching total "switchable" fuel consumed (all liquids + natural gas + coal + electricity) are the same.

$$deltaqty(f) = \sum_{l \in F} \eta_{f,l} \left(\frac{\Delta p_l}{p_l}\right) \quad (21)$$

The fuel mix change is used to adjust energy consumption by fuel

$$EFmQty(f,x,r,y) = EFmQty_1(f,x,r,y) * (1 + deltaqty(f)) \quad (22)$$

where

$deltaqty(f)$ = percent change in consumption of fuel f

$\eta_{f,l}$ = price elasticity of fuel f with respect to fuel $l \in F$

p_l = price of fuel $l \in F$

Fuel Shares and Consumption for the New Vintage

Like the fuel shares for existing capacity, the added fuel shares are initialized at the first projection year and then updated to account for trends and substitution due to changes in relative prices. The updates are computed by the same method described above for existing vintage, as shown starting on page 29.

High World Oil Price Fuel Substitution

In the High World Oil Price (HWOP) case, the level of petroleum consumption declines significantly. In the model formulation shown above, however, there are no cross-price elasticities, so there is no fuel substitution. This was not considered a particular problem for the original Reference case, because the model was "calibrated" through user judgment for each of the individual fuels. However, because high oil prices cause a lot of movement out of petroleum fuels, it was recognized that some of this change is a behavioral change in which less energy is consumed, but some change is a structural change where other fuels have been substituted for petroleum. A simple algorithm was built into the model to represent this fuel substitution.

In the HWOP case, a portion of the decline in petroleum consumption from the level in the Reference case is replaced by an increase in consumption of other fuels. In order to determine how much petroleum consumption has declined from the Reference case, it is first necessary to read in some data that specify the level of petroleum consumption in the Reference case. These data are read in along with some other data that provide the fractional increment of the petroleum that will be assumed to be replaced by other fuels. In the input file, the fractional increment is set to be 0.5 for all regions, meaning that 50 percent of the petroleum decrease in the HWOP case will be replaced by an increase in consumption of other fuels. The model projects the substitution over the years 2015 to 2050 and modifies the fractional increment so that it starts at 0 in 2015 and ramps up to its full value in 2019, then remains the same to 2050, after which the model solves as it does in the Reference case.

Once the model determines how much petroleum substitution to project, it allocates the amounts to natural gas, coal, and electricity based upon the current relative shares of each of these fuels. For example, if 100 trillion Btu requires a substitution, and the respective shares of natural gas, coal, and electricity are 0.4, 0.0, and 0.6, then projected natural gas consumption will increase by 40 trillion Btu, coal consumption will be unchanged, and electricity consumption will increase by 60 trillion Btu.

STEO Calibration

The U.S. Energy Information Administration's *Short-Term Energy Outlook* (*STEO*) projects worldwide liquid fuels consumption to 2013. Outside of the World Industrial Model but within the WEPS+, the Main Model reads the *STEO* data in each iteration. Based upon the results from the current iteration, the Main Model shares the total petroleum consumption to some sub-regions and to all the detail in the end use sectors. The exception to this step is the electric power sector, where liquids-fired generation is not included because electricity generation is a small liquids-consuming sector. The amount of liquids

consumption is allocated to the sectors based upon the share of liquids consumption. These quantities are then exported to the restart file so that they are available to each of the WEPS+ models for calibration to *STEO*. It is worth noting that these sector shares are performed in the Main Model for each WEPS+ iteration so that the actual amounts adapt to the changing shares of projected sector consumption as the model moves towards the equilibrium solution.

Process Flow Models for China

Calculating output by process for steel

Heretofore, all the industries in the World Industrial Model are structurally the same. In any given industry, there are two energy intensities ($NEFmIntF$ or tF), which dictate energy consumption for each industry. In particular, no allowance exists for individual *processes* within a given industry, some of which might have very different energy intensities. For example, in the iron & steel industry, crude steel is generally produced either by recycling scrap steel or by reducing iron ore into iron. Both processes lead to the same steel products but have very different energy requirements.

Allowing for a distinction between processes in the model allows the model to determine the energy use based on factors such as recycling rates. In the World Industrial Model for China, there are now process flow models for the steel and the aluminum portions of the non-ferrous metal industries. Following the method for the NEMS Industrial Demand Model (IDM)[2], the WIM now employs a simpler form of the PRODFLOW matrix similar to that in the IDM, but uses fewer processes than the IDM.

The steel category industry in the IEO contains NACE codes 24.1, 24.2, and 24.3, which include basic steel production as well as hot-rolled and cold-rolled finished products. Steel products from purchased steel is included in the IEO "other" industrial gross output category.

Following the PRODFLOW methodology in the NEMS IDM model, the "process flow" vector (z), represents physical output for a particular year. The vector z can be computed from the multiplication of a formula that includes and input output $PRODFLOW$ and a demand vector, d, which is denominated in dollars. Only the first element of d is nonzero (year subscripts are suppressed for simplicity):

$$z = (I - PRODFLOW)^{-1} d \qquad (23)$$

This matrix equation yields this system of equations (see Appendix E for the derivation):

$$qdemand = \ (total_steel_shipments) \qquad (24)$$

$$qEAF = \ P_{EAF}\ (total_steel_shipments) \qquad (25)$$

$$qBOF = \ P_{BOF}\ (total_steel_shipments) \qquad (26)$$

$$qDRI = \ P_{EAF} X_{DRI}\ (total_steel_shipments) \qquad (27)$$

[2] U.S. Energy Information Administration, Model Documentation Report: Industrial Demand Module of the National Energy Modeling System (Washington, DC, 2014)
http://www.eia.gov/forecasts/aeo/nems/documentation/industrial/pdf/m064(2014).pdf

$$coke = \ P_{BOF} X_{metcoal} \ (total_steel_shipments) \quad (28)$$

where

$qdemand$	$=$ steel production in tons
$total_steel_shipments$	= gross output of steel in tons, calculated from the macro model gross output in constant dollars and converted into physical units using an historical price parameter based on the dollar value of shipments in the steel sector and the physical quantity of steel produced
$qEAF$	= electric arc furnace steel output in tons
P_{EAF}	= share of steel output produced in an electric arc furnace
$qBOF$	= basic oxygen furnace output in tons
P_{BOF}	=share of steel output produced in a basic oxygen furnace $P_{EAF} + P_{BOF} \equiv 1$
$qDRI$	= electric arc furnace output in tons
X_{DRI}	= fraction of input into electric arc furnace that is direct reduced iron
$coke$	$=$ coke production in tons
$X_{metcoal}$	=the amount of metallurgical coal needed to produce a certain amount of coke. This element is currently zero, because metallurgical coal is not accounted for separately

In principle, these parameters can be obtained for every country from the data sources cited above, although some work will be required to compute regional parameters for WEPS+ regions. (The historical parameters P_{EAF}, P_{BOF}, and X_{DRI} will thus vary depending on the region, e.g., China has a different P_{EAF} than does India. So in principle there would be 16 different $PRODFLOW$ matrices if the process flow model methodology were to be implemented for all regions). Note that all three of these parameters will be between (or equal to) 0 and 1, and thus the matrix $(I - PRODFLOW)$ is never singular.

An important point to note is that these parameters P_{EAF}, P_{BOF}, and X_{DRI} are time dependent. ($X_{metcoal}$ equals 0 since in the *IEO2016* metallurgical coal is not accounted for separately.) Thus, if there is reason to believe that the relative fraction of electric arc furnaces will evolve over time, then it can be modified accordingly. This evolution could be a temporal trend from history or can be described using econometric-based drivers such as coal and electricity prices. For the *IEO2016* a simple linear trend that represented change in China's steel production from basic oxygen furnaces to electric arc furnaces was implemented.

Calculating fuel consumption for steel

To compute fuel consumption values ($\hat{C}_\gamma$), the resulting demand vector (the left side of the above equation) is used with an energy intensity matrix to compute actual fuel consumption values. Let fuel $\gamma \in \{MG, DS, RS, KS, LG, PC, SP, OP, NG, CL, EL, HT, WS, BM, GT, SL, OR\}$ denote a fuel (as shown in Table 7), and let C_γ denote consumption of fuel γ. Then C_γ is estimated as

$$\hat{C}_\gamma = \left(EI_\gamma^{EAF}\right)(qEAF) + \left(EI_\gamma^{BOF}\right)(qBOF) + \left(EI_\gamma^{DRI}\right)(qDRI). \quad (29)$$

Here, for example,EI_γ^{EAF} represents the energy intensity of fuels used in EAF production (in trillion Btu/tons of steel).

In general, the energy intensities EI_j^i in many cases can be estimated using data from the U.S. Energy Information Administration's Manufacturing Energy Consumption Survey (MECS) or from third party data. Some examples for specific fuels are provided. However, many of the intensities are zero: for example, $EI_{CL}^{EAF} = 0$, i.e., no coal is used in the electric arc furnace process. If the region has no DRI (or it is known that all the direct reduced iron in the country (like Iran) is made from natural gas), then it can be assumed that all coal is used in the BOF process. Note that energy intensity term EI_{CL}^{BOF} in the above equation will also incorporate the coal used in coke ovens, which is why coke is not specifically part of the fuel list.

Once the energy intensities have been estimated based on existing data, they must be normalized so that the initial energy consumption for each fuel in the steel industry equals the base year fuel consumption coming from the WEPS+ input data (which is derived from the IEA and IES data). So, for example, if the electricity consumption in steel in the base year is $C_{EL,}$ then

$$C_{EL} = N_{EL}\left[EI_{EL}^{EAF}(qEAF) + EI_{EL}^{BOF}(qBOF) + EI_{EL}^{DRI}(qDRI)\right] \quad (30)$$

The normalization factor N_{EL} is used to ensure that the above equation is an equality, and N_{EL} may be equal to 1. Each intensity is then re-scaled via the same normalization constant for that fuel:

$$SEI_{EL}^{EAF} = N_{EL}EI_{EL}^{EAF} \quad (31)$$

$$SEI_{EL}^{BOF} = N_{EL}EI_{EL}^{BOF} \quad (32)$$

$$SEI_{EL}^{DRI} = N_{EL}EI_{EL}^{DRI} \quad (33)$$

where SEI_γ^p= Scaled energy intensity for process p and fuel γ.

Over the projection period the projected fuel consumption will change as the P_{EAF}, P_{BOF}, and X_{DRI} parameters are allowed to vary over time, e.g., China is assumed to move away from BOF production towards more EAF production eventually. Each EI_j^i is a first cut and can be allowed to evolve via the Technology Possibility Curve (TPC) that is already in place for the steel industry (used for all fuels).

(Note: coke production, whether it is merchant or captive (i.e., within the steel producing facility), in terms of energy use is associated with the steel industry in the IEO. This association of coke production

with the steel industry is not completely accurate since merchant coke production (i.e., coke made in a location other than where the blast furnace is located) is associated with NACE code 19 (coke and refined petroleum products). There is no way to split captive and merchant coke production apart given the aggregations the WEPS+ model has regarding industrial classifications. Thus there is an inconsistency (merchant coke oven *energy consumption* accounted for in steel, but merchant coke *gross output* accounted for in coke, refining and petroleum products) but the ultimate effect on energy consumption should be relatively small.)

Calculating output for Non-Ferrous metals

This process flow methodology was also used for China's aluminum industry (the major component of the "non-ferrous metals" industry category). Non-ferrous metals other than aluminum are split out from aluminum based on United States Geological Service reports, which have data for Chinese production of aluminum and other non-ferrous metals detailed separately. The methodology is primarily the same as with the steel sector process flow model.

Refinery Fuel Use

For the *IEO2016,* refinery fuel use was explicitly computed in the WEPS+ Refinery Model. This is in contrast to previous reports where projected gross output shipments from the macro model were used to determine refinery fuel consumption. Because the refinery fuel consumption is directly correlated with overall liquids demand, for the *IEO2016,* liquids demand from all demand sectors (industrial, transportation, commercial, and residential) for all liquid fuels was used by the WEPS+ Refinery model to calculate the requisite fuel consumption for refinery processing. See the WEPS+ Refinery model documentation for more details.

Additional Calibrations

As noted, the base year 2012 for the *IEO2016* is also the last historical year, meaning that all subsequent years are projection years, including 2013 and 2014. One of the largest calibrations performed for the *IEO2016* was the inclusion of an "unspecified sector" in the demand modeling, which represented an adjustment for liquid fuel consumption. This adjustment was based on the apparent historical discrepancy between the data used from the *IEO*'s traditional data sources (i.e., EIA's International Energy Statistics (IES) consumption data and the IEA's historical disaggregated consumption data) and "bottom-up" analysis based on historical industrial production/gross output shipment levels. This adjustment for historical liquids fuel consumption was most pronounced for China, where implied diesel fuel intensities based on construction and mining shipments seemed much too low. Also, petroleum coke use (in the form of catalytic coke) in the refining sector in China is not accounted for by the IEA or IES, and thus an adjustment for this historical data point had to be made as well. More detail on the broader methodology and rationale for implementing this "unspecified sector" in the *IEO2016* can be found in the publication itself.

Another calibration for China that was implemented for the *IEO2016* was allowing for an explicit switch from diesel to liquefied natural gas (LNG) in heavy off-road equipment used in the mining and construction industries. Using a predetermined elasticity, the diesel – natural gas price differential at any given year in the projection period is compared to the price differential in the first projection year (2013). If the current differential is greater than the base year differential, then a portion of the

incremental diesel fuel demand for that year is assumed to be natural gas instead of diesel. The exact portion depends on the size of the price differential and the aforementioned predetermined elasticity.

Finally, a calibration for China's chemical feedstock consumption was implemented. First, based on outside third party data from IHS, coal feedstock consumption levels were hardcoded inside the WIM itself through year 2018. Post 2018, incremental feedstock use in China was projected using a preset share amongst the following chemical feedstocks: LPG (10%), naphtha (30%), natural gas (30%), and coal (30%). For future *IEO* reports, these percentages can be modified based on relative prices and/or policy changes.

Appendix A. Model Abstract

Model Name:
World Industrial Model of the World Energy Projection System Plus

Model Acronym:
WIM

Model Description:
The World Industrial Model of the World Energy Projection System Plus (WEPS+) is a computer-based energy demand modeling system of the world industrial sector at a regional level. The WEPS+ World Industrial Model for the *IEO2016* projects energy use that is consumed in the manufacturing, agricultural, construction, and mining sectors. The model projects industrial sector consumption for 18 energy sources in each of the 16 WEPS+ regions on an annual basis to 2050.

Model Purpose:
As a component of the World Energy Projection System Plus (WEPS+) integrated modeling system, WIM generates long-term projections of industrial sector energy consumption. As part of the system, the model provides consumption inputs for a variety of the other WEPS+ models. The model provides a tool for analysis of international industrial energy use within the WEPS+ system and can be run independently as a standalone model.

Most Recent Model Update:
May 2016

Part of Another Model:
World Energy Projection System Plus (WEPS+)

Model Interfaces:
WIM receives inputs from and provides outputs to a variety of other models in the WEPS+ system, through the common, shared interface file of the WEPS+.

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Documentation:

Energy Information Administration, U.S. Department of Energy, *World Industrial Model of the World Energy Projection System Plus: Model Documentation 2016*, DOE/EIA-M079(2016) (Washington, DC, May 2016).

Archive Information:

The model is archived as part of the World Energy Projection System Plus archive of the runs used to generate the *International Energy Outlook 2016*.

Energy System Described:

International industrial sector energy consumption.

Coverage:

Geographic: Sixteen WEPS+ regions: U.S., Canada, Mexico/Chile, OECD Europe, Japan, Australia/New Zealand, South Korea, Russia, Other non-OECD Europe and Eurasia, China, India, other non-OECD Asia, Middle East, Africa, Brazil, and other Non-OECD Americas. The following defines the multi-country regions:

- OECD Europe: This includes European Union and all of Europe except countries listed for Other non-OECD Europe and Eurasia. Israel is also included.
- Other non-OECD Europe and Eurasia: Albania, Armenia, Azerbaijan, Belarus, Bosnia and -Herzegovina, Bulgaria, Croatia, Cyprus, Fyrom (Macedonia), Georgia, Gibraltar, Kazakhstan, Kyrgyzstan, Latvia, Lithuania, Malta, Moldova, Romania, Serbia, Tajikistan, Turkmenistan, Ukraine, Uzbekistan
- Non-OECD Asia: This is all other Asia countries that are not specifically considered.
- Middle East: Bahrain, Iran, Iraq, Jordan, Kuwait, Lebanon, Oman, Qatar, Saudi Arabia, Syria, United Arab Emirates (UAE), Yemen
- Africa: All countries in the continent of Africa to include northern Africa countries such as Egypt.
- Non-OECD Americas: Central America, Caribbean and South America not elsewhere classified.

Mode: total industrial sector energy consumption.

Time Unit/Frequency: Annual, 20135 through 2050.

Modeling Features:

The World Industrial Model projects energy use in the manufacturing, agricultural, construction and mining sectors for 18 energy sources in each of the sixteen WEPS+ regions on an annual basis to 2050. The model primarily employs a stock-flow accounting framework that uses exogenously-specified output of specific industries as a proxy for productive capacity. In WIM, old capacity is retired over time to make way for new capacity. This framework allows for the modeling of changes in energy efficiency over time. Exogenously-specified energy prices impact both the mechanisms of the stock-flow process and fuel switching over time.

DOE Input Sources:

Energy Information Administration, International Energy Statistics Database, web site http://www.eia.gov/cfapps/ipdbproject/IEDIndex3.cfm# (as of May, 2015).

Energy Information Administration, *Short Term Energy Outlook* (STEO), Washington, D.C., (March 2016 release).

Non-DOE Input Sources:

International Energy Agency (IEA), *Energy Balances of OECD Countries*, Paris, 2014.

International Energy Agency (IEA), *Energy Balances of Non-OECD Countries*, Paris, 2014.

Oxford Economics, *Global Economics Model and Global Industrial Model, Oxford, England,* June 2015.

Independent Expert Reviews:

None

Computing Environment:

Hardware/Operating System: PC with Windows OS.

Language/Software Used: Intel Fortran version 11.1, 64-bit.

Run Time/Storage: Standalone model with one iteration runs in about 3-4 seconds, CPU memory is minimal, inputs/executable/outputs require less than 20MB storage.

Special Features: None.

Appendix B. Input Data and Variable Descriptions

The WIM model variables with the definition, type, and units are listed in Table 6. The types are the following: Exogenous variables that are received from other models; Endogenous variables that are defined and used in the WIM model. Since prices are for different regions, they are defined based on region specific Purchasing Power Parity (PPP) to convert prices to 2013 USA dollars by million Btu (2013$PPP/mill Btu). There are also shipments that are by region specific PPP to convert prices to 2010 USA dollars (2010 US $ PPP). Energy use is by Trillion Btu (TBtu). Energy intensity is energy use divided by shipments (TBtu/billion 2010 US $ PPP). Several variables are unitless because they are ratios, region names, functions, or type indicators (e.g., fuel types).

Table 6. WIM Variables

Variable	Filename	Definition	Type	Units
ACLIN		carbon price increment to coal	Exogenous	2013$PPP/mill Btu
ADSIN		carbon price increment to distillate	Exogenous	2013$PPP/mill Btu
AELIN		carbon price increment to electricity	Exogenous	2013$PPP/mill Btu
AFmInt		added since initial year intensity at fuel max detail level	Endogenous	TBtu/billion 2010 US $ PPP
AFmQty		added since initial year consumption at fuel max detail level.	Endogenous	TBtu
AFQtyA		added fuel that is not switched	Endogenous	TBtu
AFrInt		added since initial year intensity at fuel report detail level.	Endogenous	TBtu/billion 2010 US $ PPP
AFrQty		added since initial year consumption at fuel report detail level.	Endogenous	TBtu
AFShr		added fuel share	Endogenous	Unitless
AFTotA		added fuel total a (fuel consumption that cannot be switched)	Endogenous	TBtu
AGrOut		added gross output from previous year	Endogenous	Billion 2010 US $ PPP
AHTIN		carbon price increment to heat	Exogenous	2013$PPP/mill Btu
AKSIN		carbon price increment to kerosene	Exogenous	2013$PPP/mill Btu
ALGIN		carbon price increment to LPG	Exogenous	2013$PPP/mill Btu
AMGIN		carbon price increment to motor gasoline	Exogenous	2013$PPP/mill Btu
ANGIN		carbon price increment to natural gas	Exogenous	2013$PPP/mill Btu
AOPIN		carbon price increment to other petroleum	Exogenous	2013$PPP/mill Btu
ARSIN		carbon price increment to residual	Exogenous	2013$PPP/mill Btu
Bfac		benchmark factor	Endogenous	Unitless
CalibDet	WIMInput.xml	calibration detail read	Exogenous	Unitless
Ccoal		industrial coking coal consumption	Endogenous	TBtu
CCRat		ratio of industrial coking coal consumption to total industrial use of coal	Endogenous	Unitless
CHISNGQty	ChinaSngleo2015.xml	China factors for coal input, natural gas input, and natural gas output read.	Exogenous	Tons, meters, Qbtu
CrudeShr		share of petroleum used in each region that is crude oil	Endogenous	Unitless
DoFeed		do feedstock calculation by fuel for chemicals industry	Endogenous	Unitless
DoNGas		do natural gas	Endogenous	Unitless
DoPElas	WIMInput.xml	do price elasticity read	Exogenous	Unitless
DtoPFrac		dollar to physical unit fraction	Exogenous	Unitless

Variable	Filename	Definition	Type	Units
DtoPGR	WIMInput.xml	dollar to physical unit growth rate read	Exogenous	Unitless
EFmInt		existing/remaining from initial year intensity at fuel max detail level.	Endogenous	TBtu/billion 2010 US $ PPP
EFmQty		existing/remaining from initial year consumption at fuel max detail level.	Endogenous	TBtu
EFQtyA		existing fuel quantity A (fuel consumption that cannot be switched)	Endogenous	TBtu
EFrInt		existing/remaining from initial year intensity at fuel report detail level.	Endogenous	TBtu/billion 2010 US $ PPP
EFrQty		existing/remaining from initial year consumption at fuel report detail level	Endogenous	TBtu
EFShr		existing fuel share	Endogenous	Unitless
EFTotA		existing fuel total A	Endogenous	TBtu
EGrOut		existing/remaining from initial year real gross output	Endogenous	Billion 2010 US $ PPP
EGrOutA		copy of existing/remaining from initial year real gross output in case some of the “other” is idled.	Endogenous	Billion 2010 US $ PPP
FBMap		fuel type mapping function	Endogenous	Unitless
FEMap		fuel type mapping function	Endogenous	Unitless
FFmFeed		Projected consumption of feedstock in chemical sector at maximum fuel detail level	Endogenous	TBtu
GDP_PPP		GDP purchasing power parity	Exogenous	Billion 2010 US $ PPP
GDPElas	IndInput-2010_07_25_JH.xml	GDP elasticity read	Exogenous	Unitless
GDPLag	IndInput-2010_07_25_JH.xml	GDP lag read	Exogenous	Unitless
HFmFeed		historical feedstock consumption in chemicals at max fuel detail level	Exogenous	TBtu
HFmQty		historical consumption at fuel max detail	Exogenous	TBtu
HFrFeed		historical consumption in chemicals at reporting fuel detail level	Exogenous	TBtu
HFrQty		historical consumption at fuel reporting detail	Exogenous	TBtu
HQCoke		historical coking coal consumption in iron and steel	Exogenous	TBtu
HQtyOrg	ind_subsector_con.txt	historical quantity original read	Exogenous	TBtu
HSysQty		historical system quantity	Exogenous	TBtu
IGrOut		idled capacity/gross output	Endogenous	Unitless
IndGO		total gross output that is input into the model	Exogenous	Billion 2010 US $ PPP
InMacGO		dollars of output	Endogenous	Billion 2010 US $ PPP
IPIdx		industrial price index	Endogenous	Unitless
LGFDRat		ratio of LPG feedstock consumption to total industrial consumption of LPG	Endogenous	Unitless
LGFeed		industrial LPG feedstock consumption	Endogenous	TBtu
NewReq		new capacity required	Endogenous	Billion 2010 US $ PPP
NFmInt		new this year intensity at fuel max detail level.	Endogenous	TBtu/billion 2010 US $ PPP
NFmQty		new this year consumption at fuel max detail level.	Endogenous	TBtu
NFrInt		new this year intensity at fuel report detail level.	Endogenous	TBtu/billion 2010 US $ PPP
NFrQty		new this year consumption at fuel report detail level.	Endogenous	TBtu

Variable	Filename	Definition	Type	Units
NGFDRat		ratio of natural gas feedstock to total industrial consumption of natural gas	Endogenous	Unitless
NGFeed		industrial natural gas feedstock consumption	Endogenous	TBtu
NGrOut		new this year real gross output	Endogenous	Billion 2010 US $ PPP
OPFDRat		ratio of other petroleum feedstock to total industrial use of sequestered petroleum	Endogenous	Unitless
OPFeed		industrial other petroleum feedstock consumption	Endogenous	TBtu
PCLIN		price of coal over forecast period	Exogenous	2013$PPP/mill Btu
PDSIN		price of distillate fuel over forecast period	Exogenous	2013$PPP/mill Btu
PEAFac		TPC adjustment factor with respect to price for the new vintage	Endogenous	Unitless
PELIN		price of electricity over forecast period	Exogenous	2013$PPP/mill Btu
PERFac		TPC adjustment factor with respect to price for the existing vintage	Endogenous	Unitless
PHTIN		price of heat over forecast period	Exogenous	2013$PPP/mill Btu
PKSIN		price of kerosene over forecast period	Exogenous	2013$PPP/mill Btu
PLGIN		price of LPG over forecast period	Exogenous	2013$PPP/mill Btu
PMGIN		price of gasoline over forecast period	Exogenous	2013$PPP/mill Btu
PNGIN		price of natural gas over forecast period	Exogenous	2013$PPP/mill Btu
POPIN		price of other petroleum over forecast period	Exogenous	2013$PPP/mill Btu
PrcElas	IndInput-2010_07_25_JH.xml	price elasticity read	Exogenous	Unitless
PrcIdx		price index	Exogenous	Unitless
PrcInd		price industrial	Exogenous	2013$PPP/mill Btu
PrcLag	IndInput-2010_07_25_JH.xml	price lag read	Exogenous	Unitless
PRSIN		price of residual over forecast period	Exogenous	2013$PPP/mill Btu
QCLCCIN		industrial coking coal consumption	Endogenous	TBtu
QCLIN		quantity of coal consumed for industrial purposes	Endogenous	TBtu
QHBMIN		quantity of industrial use of biomass	Endogenous	TBtu
QHCDIN		quantity of industrial use of crude oil	Endogenous	TBtu
QHCLIN		quantity of industrial use of coal including coking coal	Endogenous	TBtu
QHDSIN		quantity of industrial use of distillate	Endogenous	TBtu
QHELIN		quantity of industrial use of electricity	Endogenous	TBtu
QHGTIN		quantity of industrial use of geothermal	Endogenous	TBtu
QHHTIN		quantity of industrial use of heat	Endogenous	TBtu
QHKSIN		quantity of industrial use of kerosene	Endogenous	TBtu
QHLGIN		quantity of industrial use of LPG including feedstocks	Endogenous	TBtu
QHMGIN		quantity of industrial use of motor gasoline	Endogenous	TBtu
QHNGIN		quantity of industrial use of natural gas including feedstocks	Endogenous	TBtu
QHOPIN		quantity of industrial use of other petroleum including feedstocks	Endogenous	TBtu
QHORIN		quantity of industrial use of other renewables	Endogenous	TBtu
QHPCIN		quantity of industrial use of petroleum coke	Endogenous	TBtu
QHRSIN		quantity of industrial use of residual	Endogenous	TBtu
QHSLIN		quantity of industrial use of solar	Endogenous	TBtu
QHSPIN		quantity of industrial use of sequestered petroleum	Endogenous	TBtu
QHWSIN		quantity of industrial use of waste	Endogenous	TBtu

Variable	Filename	Definition	Type	Units
QLGFDIN		industrial LPG feedstock consumption	Endogenous	TBtu
QLGIN		quantity of LPG consumed for industrial purposes	Endogenous	TBtu
QNGFDIN		industrial natural gas feedstock consumption	Endogenous	TBtu
QNGGLIN		industrial natural gas GTL fuel consumption	Endogenous	TBtu
QNGIN		quantity of natural gas consumed for industrial purposes	Endogenous	TBtu
QNGLGIN		industrial gas LNG fuel consumption	Endogenous	TBtu
QOPFDIN		industrial other petroleum feedstock consumption	Endogenous	TBtu
QOPIN		quantity of "other" petroleum consumed for industrial purposes	Endogenous	TBtu
REINew		relative energy intensity for new capital read WIMInput.xml	Exogenous	Unitless
RetElas	WIMInput.xml	retirement elasticity read	Exogenous	Unitless
RetRate	WIMInput.xml	retirement rate read	Exogenous	Unitless
RLLab		stores labels for regions	Endogenous	Unitless
RSLab		provides short names for each region	Endogenous	Unitless
TFmInt		Total intensity at fuel max detail level.	Endogenous	TBtu/billion 2010 US $ PPP
TFmQty		Total consumption at fuel max detail level.	Endogenous	TBtu
TFrInt		Total intensity at fuel report detail level.	Endogenous	TBtu/billion 2010 US $ PPP
TFrQty		Total consumption at fuel report detail level.	Endogenous	TBtu
TGrOut		total real gross output	Endogenous	Billion 2010 US $ PPP
TPCAddElas	WIMInput.xml	technology possibility curve adding elasticity read	Exogenous	Unitless
TPCNewGR	WIMInput.xml	technology possibility curve new growth rate read	Exogenous	Unitless
TPCRemElas	WIMInput.xml	technology possibility curve removal elasticity read	Exogenous	Unitless
TPCRemGR	WIMInput.xml	technology possibility curve removal growth rate read	Exogenous	Unitless
Tqty		total quantity	Endogenous	
TrendGr	IndInput-2010_07_25_JH.xml	trend growth rate read	Exogenous	Unitless
WAPrc		weighted average price	Endogenous	2013$PPP/mill Btu
WOPCase		world oil price case run	Endogenous	Unitless
ZBMIN		quantity of biomass consumption over forecast period	Endogenous	TBtu
ZCDIN		quantity of industrial consumption of crude oil over forecast period	Endogenous	TBtu
ZCLCCIN		industrial coking coal consumption	Endogenous	TBtu
ZCLIN		quantity of coal consumed for industrial purposes over forecast period	Endogenous	TBtu
ZDSIN		quantity of industrial consumption of distillate over forecast period	Endogenous	TBtu
ZELIN		quantity of electricity consumption over forecast period	Endogenous	TBtu
ZGTIN		quantity of geothermal consumption over forecast period	Endogenous	TBtu
ZHTIN		quantity of heat consumption over forecast period	Endogenous	TBtu
ZKSIN		quantity of industrial consumption of kerosene over forecast period	Endogenous	TBtu

Variable	Filename	Definition	Type	Units
ZLGFDIN		industrial coal gasification	Endogenous	TBtu
ZLGIN		quantity of industrial consumption of LPG over forecast period	Endogenous	TBtu
ZMGIN		quantity of industrial consumption of motor gasoline over forecast period	Endogenous	TBtu
ZNGFDIN		industrial natural gas feedstock consumption	Endogenous	TBtu
ZNGIN		Quantity of industrial consumption of natural gas over forecast period	Endogenous	TBtu
ZNGLPIN		industrial natural gas lease and plant consumption	Endogenous	TBtu
ZOPFDIN		industrial other petroleum feedstock consumption	Endogenous	TBtu
ZOPIN		quantity of industrial consumption of other petroleum over forecast period	Endogenous	TBtu
ZORIN		quantity of industrial consumption of other renewables over forecast period	Endogenous	TBtu
ZPCIN		quantity of industrial consumption of petroleum coke over forecast period	Endogenous	TBtu
ZRSIN		quantity of industrial consumption of residual over forecast period	Endogenous	TBtu
ZSLIN		quantity of industrial consumption of solar over forecast period	Endogenous	TBtu
ZSPIN		quantity of industrial consumption of sequestered petroleum over forecast period	Endogenous	TBtu
ZWSIN		quantity of waste consumption over forecast period	Endogenous	TBtu

Table 7. Fuel Abbreviations

Abbreviation	Fuel Name
MG	Motor Gasoline
DS	Distillate
RS	Residual
KS	Kerosene
LG	Liquefied Petroleum Gas (LPG)
PC	Petroleum Coke
SP	Sequestered Petroleum
OP	Other Petroleum
NG	Natural Gas
CL	Coal
EL	Electricity
HT	Heat
WS	Waste
BM	Biomass
GT	Geothermal
SL	Solar
OR	Other Renewables

Appendix C. References

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Appendix D. Data Quality

Introduction

For 16 regions of the world, WIM projects world industrial use of fuels (including motor gasoline, distillate fuel, residual fuel, kerosene, liquefied petroleum gas including feedstocks, petroleum coke, sequestered petroleum, other petroleum use including feedstocks, direct use crude oil, natural gas including feedstocks, coal including coking coal, electricity, heat, waste, biomass, geothermal, solar, and other renewables). The WIM projections are computed from the data elements listed in Appendix B. Chapter 4 (Model Structure) details the WIM estimation methods and the inputs required to implement the model algorithms. Here the principal sources of input data are described and information is provided, where available, about the quality of the imported parameter estimates.

Source and Quality of Input Data

Source of Input Data

- *STEO* – Short-term liquid fuel consumption from 2005 to 2013 are provided by region from EIA's *Short-Term Energy Outlook.* The *IEO2016* results are benchmarked to the STEO results.

- *International Statistics Database* – The Energy Information Administration provides historical data on international energy consumption by fuel type from 1980 through 2012. These data are used as the historical basis for all regional projections that appear in the *IEO2016*. While the numbers are continuously updated, WEPS+ used a "snap shot" of the database as it existed on April, 2015 as the source of its international data.

- *International Energy Agency* – The by-end-use-sector, by-product historical data are available from the OECD and non-OECD balances and statistics database by country on the subscription site www.iea.org. These data are benchmarked to the historical aggregate energy consumption data provided in the Energy Information Administration's international statistical data base.

- *NEMS* – Many of the assumptions about price and economic elasticities are based in large part on those included in the National Energy Modeling System for the United States. Expert judgment has, in some cases, been used to alter assumptions based on analyst knowledge about specific regions in the WEPS+ model.

Data Quality Verification

As a part of the input and editing procedure, an extensive program of edits and verifications was used, including:

- Checks of projected values for world and U.S. industrial consumption of fuels, retail prices, and elasticities, based on historical data, previous projections, responses, knowledge of regional economics, and technical knowledge

- Consistency checks

- Technical edits to detect extreme values or variability and correct errors

Appendix E. Derivation of Matrix Terms for Steel

This Appendix describes the full derivation of the PRODFLOW matrix for steel into the equation system represented on Page 31. Following the PRODFLOW methodology in the NEMS IDM model, the "process flow" vector , represents physical output for a particular year. The vector z can be computed from the multiplication of a formula that includes and input output (I/O) matrix A and a demand vector, d, which is denominated in dollars. Only the first element of d is nonzero (year subscripts are suppressed for simplicity):

$$z = (I - PRODFLOW)^{-1} d$$

where I = identity matrix, A = I/O matrix, and d = demand vector. In matrix form this equation is:

$$\begin{pmatrix} qdemand \\ qEAF \\ qBOF \\ qDRI \\ coke \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ -P_{EAF} & 1 & 0 & 0 & 0 \\ -P_{BOF} & 0 & 1 & 0 & 0 \\ 0 & -X_{DRI} & 0 & 1 & 0 \\ 0 & 0 & -X_{metcoal} & 0 & 1 \end{pmatrix}^{-1} \begin{pmatrix} total_steel_shipments \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad (34)$$

When the inverted matrix terms are explicitly represented[3],

$$\begin{pmatrix} qdemand \\ qEAF \\ qBOF \\ qDRI \\ coke \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ P_{EAF} & 1 & 0 & 0 & 0 \\ P_{BOF} & 0 & 1 & 0 & 0 \\ P_{EAF} X_{DRI} & X_{DRI} & 0 & 1 & 0 \\ P_{BOF} X_{metcoal} & 0 & X_{metcoal} & 0 & 1 \end{pmatrix} \begin{pmatrix} total_steel_shipments \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad (35)$$

which yields the system of equation as on page 31:

$$qdemand = (total_steel_shipments)$$

$$qEAF = P_{EAF}\ (total_steel_shipments)$$

$$qBOF = P_{BOF}\ (total_steel_shipments)$$

$$qDRI = P_{EAF} X_{DRI}\ (total_steel_shipments)$$

$$coke = P_{BOF} X_{metcoal}\ (total_steel_shipments)$$

[3] Source: Matrix calculator: https://matrixcalc.org/en/#%7B%7B1,0,0,0,0%7D,%7Ba,1,0,0,0%7D,%7Bb,0,1,0,0%7D,%7B0,c,0,1,0%7D,%7B0,0,d,0,1%7D%7D%5E%28-1%29

Appendix F. Derivation of Elasticities

Below is the derivation of the retire elasticity and energy intensity elasticity equations. By the standard definition of price elasticity we have

$$Elasticity = \frac{\Delta x/x}{\Delta P/P}$$

Where x is usually the price sensitive (demand) quantity but here we define it as the retirement rate or energy intensity; P of course represents the price. Allowing the finite differences to become differentials in the limit as $\Delta x \to 0$, i.e., letting $\Delta x \to dx$ and $\Delta P \to dP$ we have

$$Elasticity = \frac{dx/x}{dP/P}.$$

The natural log ($\ln x$) is defined as the integral of $1/t$ is, from 1 to x, i.e.,

$$\int_1^x \frac{dt}{t} \equiv \ln\ x$$

and so

$$\frac{dx}{x} = d[\ln x].$$

Approximating the differential on the right with a finite difference we have

$$\frac{dx}{x} \approx \Delta[\ln x].$$

If Δ is the difference operator for quantities representing adjacent time points (one year apart for the *IEO2016* model), then

$$\Delta[\ln x] = \ln x(t) - \ln x(t-1) = \ln \frac{x(t)}{x(t-1)},$$

and similarly for approximating dP/P . Thus finally we have the desired result:

$$Elasticity = \frac{\ln[x(t)/x(t-1)]}{\ln[P(t)/P(t-1)]}.$$